REPORT RESUMES

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A MODERN MATHEMATICS PROGRAM AS IT PERTAINS TO THE INTERRELATIONSHIP OF MATHEMATICAL CONTENT, TEACHING METHODS AND CLASSROOM ATMOSPHERE (THE MADISON PROJECT).

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ACTIVITIES ARE DISCUSSED WHICH TOOK PLACE DURING THE SECOND YEAR OF THE PROPOSED 5-YEAR "MADISON PROJECT," A COMPREHENSIVE EFFORT AIMED AT GIVING SCHOOL CHILDREN AN EXPERIENCE IN THE CREATIVE AND OPEN-ENDED FORM OF MATHEMATICS AS IT IS KNOWN TO MATHEMATICIANS. THE REPORT OF THE FIRST YEAR (1962) BORE THE SAME TITLE AS THIS REPORT. OBJECTIVES OF THIS "DEMONSTRATION" PHASE INCLUDED (1) WORK WITH SCHOOLS AND SCHOOL SYSTEMS IN PROVIDING STUDENTS WITH EXPERIENCES WHICH REFLECT VALID INTERPRETATION OF GOOD MATHEMATICAL THINKING, (2) EXTENSIVE COLLECTION OF DATA FOR PURPOSES OF DOCUMENTING THE NATURE OF THE EXPERIENCES, (3) "QUALITY CONTROL" OF EXPERIENCES, (4) INTEGRATION OF THE DEMONSTRATION INTO A PROGRAM OF INSERVICE TEACHER TRAINING, PRESERVICE TEACHER EDUCATION, AND CURRICULUM MATERIALS DEVELOPMENT, AND (5) REPORTING TO THE PROFESSION AND ACADEMIC COMMUNITY. PROCEDURES FOR THE DEMONSTRATION WERE AIMED TOWARD THE DEVELOPMENT OF A COMBINATION OF VARIOUS ATTRIBUTES THAT TOGETHER WOULD SERVE TO GIVE CHILDREN EXPERIENCES IN GOOD MATHEMATICAL THINKING. CONCLUSIVE RESULTS WERE NOT REPORTED FOR THIS PHASE OF THE PROJECT. ANALYSIS OF TAPE RECORDINGS, FILMS, AND SCHOOL-ADMINISTERED ACHIEVEMENT TESTS INDICATED SOME POSITIVE FINDINGS WHICH WERE DISCUSSED IN DETAIL WITH RESPECT TO THEIR APPROPRIATENESS AND VALUE IN MATHEMATICS INSTRUCTION. (JC)

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
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A MODERN MATHEMATICS PROGRAM AS IT PERTAINS TO THE INTERRELATIONSHIP OF MATHEMATICAL CONTENT, TEACHING METHODS AND CLASSROOM ATMOSPHERE. (THE MADISON PROJECT)

Cooperative Research Project No. D-044

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1963

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ABSTRACT

A MODERN MATHEMATICS PROGRAM AS IT PERTAINS TO THE INTERRELATIONSHIP OF MATHEMATICAL CONTENT, TEACHING METHODS AND CLASSROOM ATMOSPHERE.
(THE MADISON PROJECT)

Robert B. Davis

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Cooperative Research Project No. D-044

Note

The present report covers the second year of a proposed 5 year "demonstration." The report of the first year bears the same title as the present report, but is dated 1962.

BACKGROUND

The recent past has witnessed widespread activity in relation to "new" mathematics curricula for grades K-12. The central essence of "modernization" efforts probably lies in a "new" interpretation of the nature of mathematical thought. [1] "Mathematical thought" as interpreted by mathematicians is a perishable intangible not easily taught on a broad scale; it differs markedly from the more tangible set of techniques and facts that usually received emphasis in "traditional" curricula. Considerable informal evidence points to the difficulties encountered in attempting widespread teaching of "modern" mathematical thinking, and to the consequent need for careful regulation of the mathematical experiences offered by the school. The present "demonstration" is an attempt to work with public, private, and parochial school systems, in providing for students some experiences that will reflect a valid interpretation of "good mathematical thinking."

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[1] The use of the words "new" and "modern" in relation to recent attempts to revise the mathematics curriculum is inaccurate on several counts; nonetheless the usage has become almost universal, and these words are used here for want of satisfactory alternatives. A similar situation exists in relation to the word "traditional."

OBJECTIVES

The objectives of the "demonstration" are:

1) To work with selected schools and school systems in providing students with experiences that accurately reflect a valid interpretation of "good mathematical thinking";

2) To make extensive collection of data that will serve to document, as specifically as possible, the nature of these experiences;

3) To achieve the most effective possible "quality control" in assuring the relevance and effectiveness of these experiences;

4) To integrate this "demonstration" as effectively as possible into the Madison Project's program of in-service teacher training, pre-service teacher education, and development of curriculum materials;

5) To report as effectively as possible to the profession and to the academic community: that is, to teachers, principals, supervisors, other educators, mathematicians, psychologists, scientists, board of education members, parents, etc. (It is assumed that this communication will be in the nature of a dialogue, in which the Project will receive valuable counsel from the academic community, particularly concerning the effectiveness of Project materials and the general appropriateness of the learning experiences.)

PROCEDURE

It should be recognized that the present "demonstration" is an exercise in evolution -- it is not conceived to test explicit hypotheses, but rather to develop a combination of various attributes (not fully explicit) that together will serve to give children experience in good mathematical thinking.

The procedure during 1962-1963 has consisted of the following:

1) Developing an organizational structure between the Project and the participating school systems that provides for mathematically and pedagogically competent personnel to teach or to supervise work in each participating school;

2) Arrange for Project materials to be taught to children in grades 4, 5, 6, and 7, for approximately one hour per week (although other arrangements are also used);

3) The development of Project materials has been supported by other sponsoring agencies, namely: the Marcel Holzer Foundation, the Alfred P. Sloan Foundation, the Ford Foundation, the National Science Foundation, Syracuse University, and Webster College;

4) Provide weekly or bi-weekly in-service training for teachers in participating schools (some of this activity has been sponsored by the National Science Foundation);

5) Provide more intensive training for the Project's "specialist teachers";

6) Accumulate a library of tape-recordings of actual classroom lessons;

7) Arrange for a panel of mathematicians, teachers, and psychologists to analyze selected tape-recorded lesson excerpts for purposes of an analytical identification of apparently relevant factors in the mathematical thinking, the classroom experience, and teacher effectiveness;

8) Begin the use of the panel analysis just mentioned as a means of determining the effectiveness of the classroom experiences provided for the children;

9) To accumulate a library of 16 mm. sound motion pictures recording actual classroom lessons, as a means of preserving the fullest possible record of the nature of the classroom experiences that are provided for the children (the cost of this has been met mainly by a group of industries and trade unions in greater St. Louis, and by a grant of $208,000 from the National Science Foundation;

10) Use of the films just mentioned as a means of reporting to the teaching and mathematics professions;

11) Use of the films just mentioned as a means of teacher training;

12) Extensive consultation with professional leaders as a guarantee of the relevance and appropriateness of the learning experiences provided by the Project.

RESULTS AND CONCLUSIONS

Recalling that 1962-1963 was the second year of a proposed 5 year activity, it would be premature to attempt a report of "conclusive" results at this time.

Nonetheless, analysis of tape recordings, films and school-administered achievement tests indicates that:

1) Under certain conditions a significant fraction of 4th, 5th, 6th, and 7th graders can make creative use of the mathematical ideas presented in the Madison Project materials;

2) Creative use of these mathematical ideas by the children is strongly dependent upon the nature of the experiences provided by the school;

3) At least some of the relevant aspects of these experiences can be identified by the study of tape-recorded lesson excerpts, and can be communicated in highly specific form through the medium of such tape recordings (an even more effective communication is possible by the use of films);

4) In-service teacher education can help teachers to provide children with more satisfactory experiences in creative mathematical exploration;

5) Integration of such "creative exploratory mathematical experiences" into the school program can be achieved with reasonable success, as judged by teachers and administrators;

6) Integration of such experiences into the school program shows no observable damage to other parts of the child's educational experiences, as judged from school-administered achievement tests, observations of school psychologists, and the judgment of teachers and principals;

7) The use of films as a means of recording classroom experiences appears to have many important advantages over most other methods of data recording.

It should be noted that the response of students, parents, and communities generally has been most enthusiastic. Parents regularly report that the weekly "mathematics lesson" has become a high-point of many children's school experiences. In addition, eminent mathematicians who have studied Project films are generally agreed upon the appropriateness and value of the kinds of mathematical experiences that are being provided for the children.

Various behavioural science matters that relate to the "demonstration" are considered briefly in this report.

I. CONTEXT

The present report is concerned with those activities of the Syracuse University-Webster College "Madison Project" during 1962-1963 which were sponsored by the Cooperative Research Program of the Office of Education of the United States Department of Health, Education, and Welfare. To be intelligible, however, those activities need to be viewed in relation to two broader contexts: first, in relation to the entire program of activities of the Madison Project; second, in relation to the general directions of evolution of mathematics education in America today.

Concerning the general evolutionary patterns in America, it is quite clear that the ways whereby children learn science and mathematics in our society are changing, and changing drastically. In an attempt to deal briefly with so large a subject, we shall focus upon two significant examples, both of which are projects sponsored by Educational Services, Incorporated, of Watertown, Massachusetts (an independent, non-profit organization closely associated with M.I.T. and Harvard). The first of these is the Cambridge Conference on School Mathematics, directed by Professor Andrew Gleason of Harvard University and Professor William Ted Martin of M.I.T. The second is the Elementary School Science Project, directed by Professor David Hawkins of the University of Colorado.

The Cambridge Conference on School Mathematics ("Cambridge Conference") met during the summer of 1963 at Radcliffe College. It included mathematicians, scientists, statisticians, and engineers of the highest level of competence that exists within the American academic and scientific community. The Conference was charged with the task of formulating a general outline for the evolution of school mathematics over the next three decades. The report of the Cambridge Conference has not been published at the time of this writing, but is scheduled to appear shortly.[1]

Without trying to summarize this extensive report, it is worth noting that the Conference recommends a major effort over the next 30 years to move significant amounts of modern algebra, of geometry, of science, and of logic into the school program for grades K-6; it recommends introducing calculus and linear algebra into the secondary school program; and it recommends an especial effort at teacher training in order to make all of this feasible.

Upon what can such recommendations be based? What is most relevant about this Conference is the general nature of the task with which it was faced. To plan for a rather rapid evolution extending 30 years or more into the future involves, inevitably, an intellectual groping in darkness. Possibilities, both desirable and undesirable, cannot be seen clearly, once we renounce simple repetitions of present practice. No "vocabulary of alternatives" is available, from among which we can make selections. Such rational planning as is possible must be based upon analysis, criticism, extrapolation, and interpolation of these relatively few "thrusts and probes" into the area of possible educational alternatives which have been made and reported up to the present time.

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[1] Inquiries should be addressed to: Mr. Stephen White, Educational Services, Incorporated, 37 Galen Street, Watertown 72, Massachusetts.

The Madison Project, As Seen From One Perspective

Seen in this perspective, the Madison Project might be said to have four rather broad objectives. The first of these is to make just such "thrusts and probes" as were mentioned above, and to report the results back to those educators, teachers, mathematicians, scientists, and parents who may be interested.

What happens, for example, when one tries to teach the arithmetic of signed numbers to first or second graders? The Project has made some "probes" in this direction, and has already begun the task of reporting the results to the profession, both via written materials and via 16 mm. sound motion-picture films that show actual classroom lessons. [1]

What happens when you encourage children in grades 2 through 8 to work with Cartesian co-ordinates? The Project has made an extended series of probes in this direction, and some of the results have been reported in the films A Lesson With Second Graders, Second Lesson, Circles and Parabolas, Graphing an Ellipse, and Weights and Springs; and also via various written materials.

What happens when you encourage children in grades 4 through 9 to think about algebraic identities and proofs of algebraic theorems? When you get 5th, 6th, and 7th graders thinking about the algebra of matrices? When you try to teach a rigorous theory of the limit of a sequence to 8th graders? When you let 8th graders devise their own experiments involving velocity and acceleration? When you ask 9th graders to make up a geometry based upon vectors?

The Project has probed in all of these directions, and in many others, and reports the outcomes via film, tape recording, and written materials. A detailed listing of these materials is provided elsewhere in this report. [2]

From the Project's reports of these "thrusts and probes," future curriculum planners on any level -- teacher, supervisor, consultant, author, or anyone else -- can gain some

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[1] See especially the article "Math Takes a New Path" and the films Kye's Arithmetic, Education Report: The New Math, A Lesson With Second Graders, First Lesson, and Second Lesson. (cf. Bibliography at end of this report.)

[2] See pp. 47-51.

guidance concerning the generally unknown world of future education and future educational possibilities. This activity might be described as attempting to "enlarge the vocabulary" concerning possible curriculum alternatives.

Enlarging the School Repertoire

The activity described in the preceding section is not enough to produce rational evolution in the mathematics curriculum. Regarded from the point of view of schools and of teachers, it is necessary also to enlarge the "standard repertoire" with which they are familiar; teachers are generally at home with the arithmetical operations on the non-negative integers, with fractions, decimals, ratio, and per cent. They are not, however, familiar with Cartesian co-ordinates, matrix algebra, variables, functions, linearity, measurement errors (including the concepts of average, variance, inner-quartile range, etc.), vectors, identities, the roles of axioms and theorems, the arithmetic of signed numbers, and so on. A second Project objective is to move topics such as these into the standard repertoire of teachers and curriculum planners. This second objective obviously overlaps the first objective mentioned earlier, and many of the same devices -- especially the films -- are valuable in both undertakings.

One of the major attempts at "enlarging the teacher's repertoire," is the Project's new "packaged" in-service course which is described later in this report.[1]

Devising Creative Mathematical Experiences for Children

The third Project objective is in many ways the foundation upon which the first two rest: the devising of fully worked-out mathematical experiences for children.

It it not enough to deal in lists of topics; this approach is too abstract to provide an effective tool for promoting evolution. It is not enough to say that 6th grade children study the concepts of "average" and "variance." One must present far more detailed accounts of actual experiences the children can have.[2] In this example (of "average" and "variance") a fully described experience might be the following:[3]

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[1] See p. 36.

[2] Cf. Paul R. Klohr and Jack R. Frymier, "Curriculum Development: Dynamics of Change" (Chapter VIII) Review of Educational Research, Vol. XXXIII, No. 3, June 1963, p. 304.

Hilda Taba, Curriculum Development: Theory and Practice, New York: Harcourt, Brace, and World, 1962.

William M. Alexander, "Curriculum Planning and Development: A Review of Reviews" (Chapter X), Review of Educational Research. Vol. XXXIII, No. 3, June 1963, p. 332.

Jerome S. Bruner, The Process of Education, Cambridge, Massachusetts: Harvard University Press, 1963.

[3] Indeed, precisely the experience described here is shown as an actual classroom lesson with 6th grade children in the film entitled Average and Variance.

> In a class of about 30 children, 4 children guess (independently) the length of the classroom. For these 4 estimates, an average and variance is computed. Next, 4 children (independently) measure the length, using six-inch plastic rulers, and for these 4 estimates an average and variance is computed. Next, 4 children independently measure the length of the classroom using yardsticks, and again an average and a variance is computed for these 4 figures. Finally, 4 independent teams of children make the same measurement, using a surveyor's tape measure, and an average and variance is computed. The roles of the average and of the variance are discussed throughout; at the end, the children are asked how they would get the "right" or "exact" answer.

Emphasis on "Informal Exploratory Experiences"

Even as detailed an "experience" as that listed above is not sufficiently explicit. Although it deals reasonably completely with the abstract mathematical content of the classroom activity, it does not deal with the "tone" or "atmosphere" of the class. Many of the "new curriculum" projects in mathematics and science have come to rely on informal, exploratory experiences, rather than on the pedantic, academic, formal exposition which has been commonly associated with the study of science and mathematics in the past. The list would include the Elementary School Science Project mentioned earlier, the University of Illinois Arithmetic Project (directed by Professor David Page), and the Madison Project.

The delineation of the "tone" of these informal exploratory experiences becomes an important matter. What is especially important is that one is concerned here with the task of changing attitudes, since informal creative experiences of any sort are not what most adults think of in relation to "mathematics" and "science." Many teachers, in particular, must look at these activities from a point of view quite different from the one they have habitually used, if they are to experience success with this new approach.

Here again, films showing actual classroom lessons play a central role.

It is worth remarking that the development of the various "informal exploratory experiences" must, of course, occur in actual classrooms, with living children. So great is the importance of this aspect that all three projects mentioned above commonly speak of their materials as "being written by the children themselves." This is obviously an exaggeration, but it points to a significant truth.

It also points to the need for a collaboration between the Project and various schools, public, private, or religious. This collaboration, specifically, is the portion of Project activities that is supported by the United States Office of Education.

Each individual "informal exploratory experience" is to some extent an entity in its own right, but in the life of the child they do not stand apart. One must pay special heed to the cumulative effect of a long and carefully devised sequence of such experiences, extending indeed over several years of the child's life. Moreover, it is important to consider also the task of integrating these experiences into the general school program.

Toward A "Theory of Instruction"

All of the preceding activities, of course, stand on a foundation of attitudes, values, and working assumptions, both explicit and implicit. The fourth Project objective is to make all of this undergirding as explicit as possible, in order to contribute to what Bruner has called[1] a "theory of instruction."

These, then, are the four "strategic" objectives of the Project:

1. To make "probes" and "thrusts" into the unknown world of possible mathematical experiences of children (such as "what happens when you teach signed numbers to first graders") and to report the results, especially via motion picture films showing actual classroom lessons. This has the effect of increasing the "vocabulary of possible alternatives" available to educational planners at all levels.
2. To move new topics into the working repertoire of teachers, schools, and textbook authors.
3. To devise "informal exploratory experiences," which are fully designed (but flexible) classroom experiences for children (these are, in effect, what the films really show).
4. To formulate explicitly the working hypotheses and value judgments upon which the preceding activities are based, in order to contribute to the development of a "theory of instruction."

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[1] Bruner, *ibid*.

The portion of Project activity supported by the U.S. Office of Education is the actual collaboration with 14 school systems in building in each school system a functioning, high-quality "modern" mathematics program in which Project-devised "informal exploratory experiences" play a major role.

II. ORGANIZATIONAL STRUCTURE, PROCEDURE, AND PERSONNEL

The Madison Project maintains offices at Syracuse University, at Webster College, and at Weston, Connecticut.[1] The Project Director is Professor Robert B. Davis, of Syracuse University and Webster College. Project personnel (and affiliated personnel in participating schools) for '62-'63 included:

Associate Director: Katharine Kharas

Senior Research Associate: Beryl Cochran

Co-ordinator for Syracuse University: Professor Donald E. Kibbey

Co-ordinator for Webster College: Sister M. Jacqueline, S.L.

Film Production: Morton Schindel

Specialist Teachers: Betty Bjork
Marianne Heidt Ockerbloom
Dante Zacavish
Donald Hubbard
Sophie Ciola
Jane Marlatt
William Wilson

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[1] The complete addresses are:

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Syracuse University
Syracuse 10, New York
Telephone No. GR 6-5571

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Arthur Scheid
Marilyn de Santa
Lyn McLane
Donald Cohen
Cornell Pierce
Edith McKinnon

Psychologists: Herbert Barrett
Carl Pitts

Consultants (Psychology): Professor Jerome Bruner, Harvard University
Professor Richard de Charms, Washington University
Dr. Jerome Kagan, Fels Research Institute, Antioch
Dr. C. Brooks Fry, M.D. (child psychiatry)
Dr. Carol Fry, M.D. (child psychiatry)

Consultant (Physical Science): William Walton

Consultants (Education): Cynthia Parsons
Leonard Sealey

In addition, the Project owes a deep debt to many teachers, principals, curriculum directors, school superintendents, mathematicians, psychiatrists, and psychologists, not listed above, who gave their time with surprising generosity in so busy an age.

The Project also has a functioning staff of administrative assistants, secretaries, film processing personnel, etc. Videotaping and kinescoping facilities and services are contracted through Weston Woods Studios.

The Role of Specialist Teachers: Specialist teachers teach previously developed "informal exploratory experience" units in participating schools, help to develop new units, work with classroom teachers individually, operate in-service and summer courses for teacher education, and represent the Project at professional meetings. In addition, the specialist teachers meet together in order to study tape-recorded classroom lessons, or films, and to collaborate in the maintenance of high-quality learning situations. Although academic freedom is a goal never to be lost sight of, the specialist teachers during the period of their affiliation with the Project make a considerable effort to develop a commonly-shared teaching style.

In the first phase of operation, the "informal exploratory experiences" are regarded as enrichment, and become an addition to the usual school curriculum.

What is involved here is an exercise in evolution, and "evolution by consent" -- the specialist teacher works in a given class only upon the invitation of the regular classroom teacher. Wherever the classroom teacher ultimately feels prepared to do so, he may take over the teaching of the Madison Project materials, and may ultimately dispense with the specialist teacher entirely. The specialist is prepared to work with the classroom teacher on the task of integrating the "informal exploratory experiences" into the regular arithmetic and science programs wherever feasible.

Curriculum decisions remain, as always, with the classroom teacher and the school, but Project personnel are available as consultants.

The main difficulties stem from the fact that it is the essential creative spirit of the "informal exploratory experiences" that is at stake here. Mathematical facts are easily transmitted, but a creative approach is highly perishable. It surely cannot become a pedagogical tool of the teacher who is unsympathetic to such an approach.

Obviously, it is the ultimate intent that the point of view of the "informal exploratory experiences" should suffuse a very large part of the entire school program, even outside of mathematics and science.

The Scarsdale Meeting: In the spring of 1963, under the leadership of Dr. Harold Howe, Dr. Thomas Aquila, and Dr. Earle Helmer, a meeting of superintendents, principals, and specialist teachers was held in Scarsdale, New York, to make plans for 1963-64. It was agreed at this meeting that the Project - school collaboration should now enter phase 2, with emphasis on integrating the "informal exploratory experiences" into the total curriculum, and with greater Project participation in school curriculum planning. Final curriculum and teaching decisions will of course remain with the classroom teachers and the participating schools. The Project's role will continue to be advisory.

Extent of School - Project Collaboration: This report is concerned with the second year of a proposed five-year "demonstration," and no formal data is presented at this point. Nonetheless, in order to give some idea of the extent of present school-Project collaboration, the following table shows an informal report on 175 classes of students in public, private, and parochial schools who have been working with presently developed "informal exploratory experiences" at grade levels 1 through 8. Preliminary developmental work on new "informal exploratory" units for nursery school, kindergarten, and grades 9 and 10 is not reported in this table. Fourteen different schools or school systems are represented in this report. All are in suburban or "exurban" communities, ranging in socio-economic status from somewhat above average to quite markedly so. (Plans for '63-'64 and subsequent years call for the inclusion of some big city "inner city" schools in markedly below average socio-economic areas.)

Notice that the numerical entries in the table indicate the number of classes to which each row-and-column description is applicable.

It can be seen that, as a general picture, one can think of Madison Project materials during phase one as being taught once a week, for a lesson about 30-40 minutes long, at the 4th grade, 5th grade, or 6th grade level, and regarded as enrichment material of an algebraic or geometric nature, largely separate from the school's pre-existing arithmetic program.

Explanation of abbreviations in table:

Item 14. The explicit meanings of these various mathematical topics are given in some detail in Section IV below.

Item 18. Obviously "team teaching" has come to have many different meanings. In answering Question 18, the official description which each school gives for its own method of operation was the basis for the present tabulation.

Item 23 (and elsewhere). "SMSG" refers to the School Mathematics Study Group, directed by Professor E.G. Begle of Stanford University. "Addison-Wesley" refers to the arithmetic and algebra series of textbooks published by the Addison-Wesley Publishing Company of Reading, Massachusetts.

Item 26. "Cuisenaire" refers to the set of colored wooden blocks used in arithmetical study, sold by Cuisenaire Corporation of America.

"GCMP" refers to the mathematics materials developed by the Greater Cleveland Project, published by Science Research Associates.

"Page" refers to the mathematical materials developed by Professor David Page, of the University of Illinois and E.S.I.

"EBF" refers to the Mathematics Workshop materials, developed by Botel, Wirtz, and Sawyer, and available from Encyclopedia Brittanica Films, Inc.

"SMSG" refers to School Mathematics Study Group materials (Stanford University).

"Dienes MAB" refers to the "multi-base arithmetical blocks" developed by Professor Z.P. Dienes of the University of Adelaide, Australia and widely used in Leicestershire County, England.

"Suppes' Geometry" refers to geometric materials developed by Professors Patrick Suppes and Newton Hawley, of Stanford University.

Item 28. "E.S.S." refers to the Elementary School Science Program of E.S.I., which is discussed earlier in this report.

"P.S.S.C." refers to the Physical Science Study Committee materials, developed by E.S.I.

Nursery School No regular classes -- exploratory phase only

Kindergarten No regular classes -- exploratory phase only

1	2	3					
Grade Level	Total Number of Classes	Homogeneous Low	Homogeneous Average	Homogeneous High	Heterogeneous	Grades in K-3	K-
1	3				3		
1 and 2	1				1		
2	4			2	2	1	
2 and 3	1				1		
3	11		1	3	7		
4	60	2		6	52		1
5	37	2	5	11	19		
6	42	8	2	16	16		
4, 5, and 6	3	1	1	1			
5 and 6	8	2	2	4			
7	3			3			
8	2			2			

Entries in table indicate number of classes to whom description applies.

se only

se only

4

Homogeneous High	Heterogeneous	Grades in this school building K-3	K-4	K-5	K-6	K-8	4-8	5-8	6-8	5th only	7th only	7-9
	3				3							
	1				1							
2	2	1			3							
	1				1							
3	7		2		9							
6	52		18	3	38		1					
11	19			2	25		1	5		4		
16	16				32	1	5	3	1			
1					3							
4					1		7					
3											3	
2								1				1

plies.

Grade Level	Madison Project Material Taught By:					
	Specialist Only	Classroom Teacher Only	Specialist and Classroom Teacher Both	30 min.	35-40 min.	41-45
1	2		1	2		
1 and 2	1			1		
2	2		2	3		
2 and 3			1	1		
3	7	1	3	6	3	
4	19	6	35	8	9	24
5	18	4	15	3	18	5
6	29	5	8	2	26	10
4, 5, and 6	3					3
5 and 6	1		7			8
7			3			
8	1	1				

6

	Length of Madison Project Lesson						
assroom Teacher th	30 min.	35-40 min.	41-45 min.	50 min.	60 min.	45-60 min. varied from lesson to lesson	Madison Project material not separated from regular arithmetic instruction
	2					1	
	1						
	3					1	
	1						
	6	3				2	
	8	9	24		1	1	17
	3	18	5	5	2	4	
	2	26	10	1	2	1	
			3				
			8				
				3			

Number of times per week that Madison Projec

Grade Level	Once a week	Once or Twice Varied -- not rigidly determined	Twice	Twice every other week	Madison Pro closely inte regular arit separation
1	3				
1 and 2	1				
2	4				
2 and 3	1				
3	10	1			
4	41	1	1		1
5	35	2			
6	40	1			
4, 5, and 6	3				
5 and 6	8				
7					
8	1			1	

7

Number of times per week that Madison Project material is taught

ice	Twice every other week	Madison Project material closely interwoven with regular arithmetic -- no separation	Irregular Schedule	Five times per week (every day)
1		17		
			1	
				3
	1			

8		9				10				
I.Q. data on class i) which test was used? ii) range iii) median	Grade level	Number of weeks of teaching Madison Project materials during '62-'63 school year				Previous background of students: Number of years of previous study of Madison Project materials				
		20 or less	21-30	31-40	over 40	0	1	2	3	4
Varies from class to class -- not reported here beyond designation of classes as Homogeneous (Low, Average, High) and Heterogeneous, in column 3.	1	2	1			3				
	1 and 2	1				1				
	2	3	1			3	1			
	2 and 3	1				1				
	3	6	4	1		10		1		
	4	12	44		4	58	1	1		
	5	4	22	1	10	31	5	1		
	6	3	32	5	2	32	8	1	1	
	4, 5, and 6		3			3				
	5 and 6	1		7		5	3			
	7			3		2	1			
	8	1	1			1			1	

	11						12							
	Background of classroom teacher: Number of years previous experience teaching Madison Project materials						Background of Specialist Teacher (if specialist teacher was involved) Number of years previous experience teaching Madison Project materials							
Grade Level	0	½	1	2	3	Unknown	0	½	1	2	3	4	5	No specialist
1	3							1		1	1			
1 and 2	1							1						
2	2		1		1			1		1	2			
2 and 3	1												1	
3	7	1	1	1	1			1	3	6	1			
4	37	1	20			2	21	4	7	27	1			
5	27	3	1	6			7		7	22	1			
6	29	2	10		1			1	4	30	1		1	5
4, 5, and 6	3							3						
5 and 6	1		7						1	7				
7				3						3				
8	1		1										1	1

13

Grade Level	Attitude of classroom teacher, as perceived and reported by specialist teacher				
	Hostile	Not very receptive	Moderately receptive, with some understanding of goals and precedures	Very receptive, with good understanding of goals and procedures	Unreported or unknown
1				3	
1 and 2				1	
2				4	
2 and 3				1	
3			2	9	
4	3	7	14	16	20
5	4	2	11	20	
6	4	4	20	14	
4, 5, and 6				3	
5 and 6				8	
7				3	
8		1		1	

14

Madison Project Topics Covered

Grade Level	Number Line	Introduction to Signed Numbers	Complete Arithmetic of Signed Numbers	Concept of Open Sentences, True and False Statements	Practice on Variables, etc., using Quadratic Equations
1	3	2		3	
1 and 2	1			1	
2	4	3	2	4	
2 and 3	1			1	
3	11	11	10	11	10
4	57	59	60	60	60
5	22	33	37	37	34
6	33	37	42	42	42
4, 5, and 6	3	3	3	3	3
5 and 6	4	8	8	8	8
7	3	3	3	3	3
8	1	1	1	1	1

Madison Project Topics Covered (continued)

Grade Level	Cartesian Coordinates	Linear Graphs	Graphs of Conic Sections	Study of Functions	Identities	Role of Axioms and Theorems
1	1					
1 and 2						
2	3	1			1	
2 and 3	1					
3	11	7	1	4	5	1
4	60	60	7	6	60	21
5	37	36	8	24	36	32
6	40	40	6	6	40	36
4, 5, and 6	3	3			3	3
5 and 6	8	8	4		8	8
7	3	3	3	3	3	3
8	1	1			1	

Grade Level	Madison Project Topics Covered (continued) Original Derivations of Algebraic Theorems	General form of Equations, and General Solution	Derivation of Geometric Mensuration Formulae	Matrices	Extension of Number Systems
1					
1 and 2					
2					
2 and 3					
3	1			1	
4	21	10		7	
5	32	9	4	14	
6	36	6	4	6	2
4, 5, and 6	3			3	
5 and 6	8	8	4		
7	3	3	3	3	3
8					1

Madison Project Topics Covered (continued)

Grade Level	Average and Variance	Empirically-Determined Functions	Velocity and Acceleration	Vectors	Limit of a Sequence
1					
1 and 2					
2					
2 and 3					
3					
4	6	2			
5	1	3	1		
6	7	4			
4, 5, and 6					
5 and 6					
7	3	3	3	3	
8			1		1

Grade Level	Madison Project Topics Covered (continued) Uses of Monotonicity	Concept of Angle	Concepts of Area, Length, Volume	Trigonometric Functions	Concept of Number (Positive Integers)
1	1				2
1 and 2					1
2	1				2
2 and 3			1		1
3	3	3	4		4
4	27	9	4		1
5	9	12	13		
6		5	4	4	
4, 5, and 6		3			
5 and 6	4				
7			3	3	
8				1	

14

Grade Level	Madison Project Topics Covered (continued)					
	Concept of Addition	Concept of Multiplication	Simultaneous Equations	Word Problems	Inequalities	Experience with Fractions
1	2					
1 and 2	1					
2	2	1				2
2 and 3	1	1				1
3	4	4			2	3
4	1	1	10	20	27	5
5			3	11	12	2
6			4	27	4	
4, 5, and 6						
5 and 6					4	
7			3	3		
8						

14

Madison Project Topics Covered (continued)

Grade Level	Graphs of Inequalities	Equivalent Equations	Similar Triangles	Implication, Contradiction, Uniqueness	Truth Tables	Inference Schemes
1						
1 and 2						
2						
2 and 3						
3	2	2		2		
4	18	18		4		
5	4	17	7	10	3	
6	4	37	4	5	2	2
4, 5, and 6						
5 and 6	4					
7		3		3	1	1
8					1	1

15

Methods of judging student progress:

Grade Level	Classroom observation only	Classroom observation plus standardized tests	Classroom observation plus teacher-made written tests	Classroom observation plus individual interviews by clinical psychologist	Classroom observation, standardized tests, and teacher-made written tests
1	2			1	
1 and 2	1				
2	2	1		1	
2 and 3	1				
3	7	2		1	
4	48	9	1	1	1
5	25	4		1	7
6	14	1	1	1	25
4, 5, and 6	3				
5 and 6					8
7			3		
8	2				

	16 Number of students in class						17 Does school cross lines at this grade	
Grade Level	Under 20	20-25	26-30	31-35	36-54	55 or more	No	Yes
1	1	1	1				2	1
1 and 2		1						1
2	1	1	2				2	2
2 and 3		1						1
3	1	2	8				7	4
4	3	2	17	37	1		57	3
5	3	10	16	8			23	14
6	2	5	19	14	1	1	41	1
4, 5, and 6		3						3
5 and 6		1		7				8
7		3					3	
8	1			1			2	

ss		17 Does school cross grade lines at this grade level?		18 Does school use team teaching at this grade level?		
36-54	55 or more	No	Yes	No	yes	Does Not Apply --Departmentalized Secondary School
		2	1	2	1	
			1		1	
		2	2	3	1	
			1		1	
		7	4	8	3	
1		57	3	40	20	
		23	14	26	11	
1	1	41	1	40	1	1
			3		3	
			8	1	7	
		3				3
		2				2

	19							20				
	Number of students who did quite well on Madison Project materials, as determined by method listed in column 15							Number of students who learned little or nothing from "informal exploratory experiences," as judged by methods listed in column 15				
Grade Level	0-3	4-6	7-9	10-15	16-20	over 20	not reported	0-3	4-6	7-9	10 or more	not reported
1					1	2		2	1			
1 and 2				1	1							1
2			1		1	2		1	2			1
2 and 3							1					1
3			2			8	1	8	1			2
4		4	10	15	8	20	3	19	32	5	1	3
5			8	8	6	11	4	13	14		6	4
6			1	14	5	21		22	14	3	3	
4, 5, and 6			1		2							3
5 and 6				7		1		8				
7					3			1	2			
8	1				1			1	1			

Grade Level	21 Student Attitude Little real interest	Moderate interest	Unusually great interest	Quite mixed	Unreported	22 Parental Attitude Hostile	Little real interest	Moderate interest	Unusually great interest	Unknown
1			3						2	1
1 and 2			1							1
2			4						3	1
2 and 3			1							1
3		2	9					6	3	2
4	4	20	35		1			6	27	27
5		11	24		2			15	21	1
6		10	32					10	31	1
4, 5, and 6			3							3
5 and 6			8						8	
7				3					3	
8			1	1					1	1

		Arrangements for following this class of childre		
Grade Level	These children will receive Madison Project instruction from regular classroom teacher	These children will receive Madison Project instruction from a specialist teacher	Will move into SMSG program next year	Wil Ad pro
1		2		
1 and 2		1		
2	1	3		
2 and 3		1		
3		10		
4	8	47		
5	11	28		
6		16	15	
4, 5, and 6		3		
5 and 6		7		
7				
8				

23

Arrangements for following this class of children next year

ren will receive ject instruction ialist teacher	Will move into SMSG program next year	Will move into Addison-Wesley program next year	Will follow selected subset	No definite plans made as yet
2				1
1				
3				
1				
10				1
47				5
28		1		6
16	15	10		1
3				
7				1
			3	
				2

	24				25		
	Relationship between Madison Project material and regular arithmetic program				Relationship between Madison Project experiences and school science program		
Grade Level	Entirely separate	Largely separate	Considerable integration	Virtually total integration	Entirely separate, no relationship	Some integration	Considerable integration
1	2			1	2	1	
1 and 2	1				1		
2	2	1		1	3	1	
2 and 3	1				1		
3	3	1	3	4	9	2	
4	15	15	25	5	54	6	
5	6	9	15	7	23	9	5
6	4	15	11	12	17	23	2
4, 5, and 6				3	3		
5 and 6		1	7		8		
7				3	3		
8	1		1		2		

What other "modern" mathematics m

Grade Level	Cuisenaire used in lower grades	Cuisenaire, GCMP, Page	Cuisenaire, Page, EBF	SMSG, Page, EBF	Dienes MAB, Page
1	1		1		1
1 and 2	1				
2	1		2	1	
2 and 3					
3	2	2	1	1	
4	24	2		2	
5	5	4			
6	2				
4, 5, and 6	3				
5 and 6					
7					
8					

26

What other "modern" mathematics materials were used

e,	SMSG, Page, EBF	Dienes MAB, Page	Page, EBF	SMSG	SMSG, Cuisenaire	GCMP, EBF, Page, SMSG
		1				
	1					
	1					
	2		5	1	3	
			2		3	5
			1	5		1
				1		

26

Grade Level	What other "modern" mathematics materials were used (continued) EBF	Suppes' Geometry	SMSG and Suppes' Geometry	Classroom teacher drew on own background	Specialist teacher drew on own background	None
1						
1 and 2						
2						
2 and 3						1
3						5
4	2	5		2	1	13
5				1	7	10
6				5	28	
4, 5, and 6						
5 and 6			7		1	
7				3	3	
8						1

What "traditional" arithmetic texts (if any)

Grade Level	Silver Burdett	Webster	Scott-Foresman and Addison-Wesley	Scott-Foresman	Ginn	Winston	Silver-Bu and Ginn
1			1				
1 and 2							
2			1				
2 and 3		1					
3	2	1	1				
4	29			1	17		
5	4	1	1		5	5	
6	1		1	1	2	1	1
4, 5, and 6							
5 and 6	1						
7							
8							

27

What "traditional" arithmetic texts (if any) were used?

t-Foresman	Ginn	Winston	Silver-Burdett and Ginn	Winston, Scott-Foresman, Harcourt, and Ginn	No fixed text used	Does not apply	Unknown
							2
							1
							3
					3		4
1	17				6		7
	5	5			15		6
1	2	1	1	5	25		5
							3
				7			
					3		
						2	

28

What other "modern" science materials were used?

Grade Level	E.S.S.	P.S.S.C.	None	Unknown
1			2	1
1 and 2				1
2			2	2
2 and 3			1	
3	2		5	4
4	2		34	24
5	4		14	9
6	1		4	37
4, 5, and 6				3
5 and 6				8
7		3		
8				2

Methods used for evaluating quality of the education experience
(Combinations are not explicitly noted here, and must be inferre

Grade Level	Frequent observation by many observers, discussion among observers	Films made showing actual classroom lessons	Tape recordings made of actual classroom lessons	Classroom tea by specialist
1	3	1	3	
1 and 2	1	1		
2	2	1	3	
2 and 3	1			
3	5	2	5	
4	8	3	5	23
5	11	2	11	12
6	17	1	5	5
4, 5, and 6	3		3	
5 and 6	8		7	
7			3	
8	2	1		

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evaluating quality of the education experience which was offered to the children.
not explicitly noted here, and must be inferred from numbers shown below)

ons	Tape recordings made of actual classroom lessons	Classroom teacher observed by specialist	Specialist taught occasional class in order to evaluate progress	Classroom teacher's own judgment
	3			
	3			
	5			
	5	23	22	4
	11	12	20	10
	5	5	23	23
	3			
	7			
	3			

30

General estimate of quality of educational experience offered to the children. Please note that, for '62-'63, this data is entirely informal. Formal data will be reported for subsequent years only.

Grade Level	Poor	Fair	Good	Excellent	Uncertain, unknown, or unreported
1			1	1	1
1 and 2					1
2				2	2
2 and 3			1		
3		2	4	4	1
4	2	4	16	6	32
5			12	13	12
6		4	7	25	6
4, 5, and 6					3
5 and 6					8
7					3
8					2

Other Project Activities: The portion of Project activity sponsored by the Cooperative Research Program of the U.S. Office of Education is the Project-school collaboration described above. Other Project activities, all closely related with the present "demonstration," include:

In-service and Summer Teacher Education Programs, some of which are sponsored by the Institutes Section of the National Science Foundation. During the summer of 1963, Madison Project instruction was offered by Project personnel at Syracuse University, Webster College, the University of Kansas City, University of Rochester, and the University of Arizona. Short introductions were presented at Harvard University and at Tulane University.

In addition, during July and August, 1963, over 8,000 orders for Project materials were filled, mainly in connection with N.S.F. Institutes at other colleges and universities.

The In-Service Package. In the past, Project personnel have taught in-service courses in the normal, face-to-face manner. The geographical limitations on this are obvious, and for this reason the Project is assembling some "packaged" materials, consisting of both films and written materials, for group self-study by elementary school teachers. Trials on this In-Service Package are scheduled to begin in the fall of 1963.

Webster College Undergraduate Program. One of the crucial issues at stake is the teacher's ability to approach mathematics and science in a tentative, open-ended way, that is perhaps well-symbolized by looking through a microscope in order to see what one can see there, rather than in order to confirm one's prejudices about what one ought to be able to see there. Science makes extreme demands upon one's flexibility and willingness to live with unresolved questions and with tentative complex answers. Traditional teacher training has often failed utterly to instill such an attitude; on the contrary, it has frequently tended to emphasize definitive answers at the price of over-simplification, arbitrariness, and outright error.

Webster College, with Ford Foundation support, is developing a special undergraduate program for prospective specialist teachers that is distinguished by learning in what Bruner has called "the inductive mode." This Webster College program is now in its third year of operation, and evidence already available strongly supports the efficacy of this approach.

Films. Under a grant of $208,000.00 from the National Science Foundation, plus additional funds from industries and labor unions in the greater St. Louis area, the Project makes 16mm. sound motion picture films showing actual classroom lessons with children experiencing "informal exploratory" activities in mathematics and in science. These films form the foundation for teacher training, for communications with mathematicians, psychologists, and educators, for reports to parents, and, indeed, for all serious reporting of Project activities. One might say that these films are an essential appendage to the present report -- but it would probably be more accurate to say that the present report, and all other written discussions and descriptions of Project activity, are appendages to the films. The main record of Project accomplishments is embodied in the collection of Project films; it cannot be otherwise.

Behavioural Science Aspects. The Project divides behavioural science aspects into four categories: the "description problem," which is concerned with describing as accurately and significantly as possible the nature of "informal exploratory experiences"; the "quality control program," which is concerned with monitoring the quality of the educational experiences being provided in the various co-operating classrooms; the "identification of working assumptions," which is concerned with identifying significant working assumptions, many of which are initially tacit; and the "psycho-social environment," which is concerned with factors mainly beyond the Project's control, such as the needs and predilections of children of different ages, or the attitude towards school held by children in a given neighborhood.

As one method of attacking the "description problem" and the "quality control problem," the Project operated, during '62-'63, the "tape panel," as follows: audio tape recordings of classroom lessons were submitted by participating schools, and became a large addition to the Project's extensive library of tape-recorded mathematics lessons. Each month 2 or 3 lessons, or portions of lessons, were selected, duplicated so that 63 identical copies were available, and these were distributed to 63 professional people who served on the 1962-63 tape panel. Membership of this panel included mathematicians, elementary school teachers, specialist teachers, secondary school science and mathematics teachers, guidance and supervisory personnel, principals, psychologists, and child psychiatrists. Each panelist wrote a detailed analysis of the lesson, and these 63 analyses were studied individually, compared, and a composite analysis was made.

The greatest value of the panel was -- as it was intended to be -- in getting consensus among the panelists, most of whom serve the Project in important roles. It served also to clarify the description of "good teaching" in relation to informal exploratory experiences. Even more important, it provides a tool for dealing with the "quality control" problem.

All of these matters will be discussed further along in this report.

III. THE NATURE OF "INFORMAL EXPLORATORY EXPERIENCES"

The central aspect of the present demonstration is, of course, the precise nature of the so-called "informal exploratory experiences." In the present section we shall consider this in some detail.

The rationale behind this particular type of mathematical experience is based upon the Project's various working assumptions. In particular, it is assumed that to know mathematical statements is not the same thing as to know mathematics.[1] Mathematical facts and techniques can be taught and learned by rote, memorization, and drill; they can be tested for quite simply; but they do not constitute mathematical understanding or mathematical wisdom. Possibly the largest single handicap to the study of mathematics during the first half of the twentieth century has been the extreme ease of transmitting mathematical facts and techniques, and the very great difficulty of distinguishing between what is only this and what is genuine mathematical wisdom.

Alfred North Whitehead has pinpointed this problem as the principle focus of his important book The Aims of Education, in which, for example, he writes:

> When you analyze in the light of experience the central task of education you find that its successful accomplishment depends on a delicate adjustment of many variable factors. The reason is that we are dealing with human minds, and not with dead matter. The evocation of curiosity, of judgment, of the power of mastering a complicated tangle of circumstances, the use of theory in giving foresight in special cases -- all these powers are not to be imparted by a set rule embodied in one schedule of examination subjects.
>
> ...With good discipline, it is always possible to pump into the minds of a class a certain quantity of inert knowledge. You take a textbook and make them learn it. So far, so good. The child then knows how to solve a quadratic equation. But what is the point of teaching a child to solve a quadratic equation? There is a traditional answer to this

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[1] cf. Bruner, *ibid.*

> question. It runs thus: The mind is an instrument, you first sharpen it, and then use it; the acquisition of the power of solving a quadratic equation is part of the process of sharpening the mind. Now there is just enough truth in this answer to have made it live through the ages. But for all its half-truth, it embodies a radical error which bids fair to stifle the genius of the modern world. ...I have no hesitation in denouncing it as one of the most fatal, erroneous, and dangerous conceptions ever introduced into the theory of education. The mind is never passive; it is a perpetual activity, delicate, receptive, responsive to stimulus. You cannot postpone its life until you have sharpened it. Whatever interest attaches to your subject-matter must be evoked here and now; whatever powers you are strengthening in the pupil, must be exercised here and now; whatever possibilities of mental life your teaching should impart, must be exhibited here and now. That is the golden rule of education, and a very difficult rule to follow.
>
> The difficulty is just this: the apprehension of general ideas, intellectual habits of mind, and pleasurable interest in mental achievement can be evoked by no form of words, however accurately adjusted. All practical teachers know that education is a patient process of the mastery of details, minute by minute, hour by hour, day by day. There is no royal road to learning through an airy path of brilliant generalizations. There is a proverb about the difficulty of seeing the wood because of the trees. That difficulty is exactly the point which I am enforcing. The problem of education is to make the pupil see the wood by means of the trees.[1]

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[1] Alfred North Whitehead, Aims of Education. (Mentor Books) New York: Macmillan Company, 1929, pp. 17-18.

Further on, in this same volume, Whitehead writes:[1]

> ...so long as we conceive intellectual education as merely consisting in the acquirement of mechanical mental aptitudes, and of formulated statements of useful truths, there can be no progress; though there will be much activity, amid aimless re-arrangement of syllabuses, in the fruitless endeavour to dodge the inevitable lack of time. We must take it as an unavoidable fact, that God has so made the world that there are more topics desirable for knowledge than any one person can possibly acquire. It is hopeless to approach the problem by the way of the enumeration of subjects which every one ought to have mastered. There are too many of them, all with excellent title-deeds. Perhaps, after all, this plethora of material is fortunate; for the world is made interesting by a delightful ignorance of important truths. What I am anxious to impress on you is that though knowledge is one chief aim of intellectual education, there is another ingredient, vaguer but greater, and more dominating in its importance. The ancients called it "wisdom." You cannot be wise without some basis of knowledge; but you may easily acquire knowledge and remain bare of wisdom.
>
> ...The ancients saw clearly -- more clearly than we do -- the necessity for dominating knowledge by wisdom. ...The only avenue towards wisdom is by freedom in the presence of knowledge.

The same distinction between a sterile knowledge of facts and techniques, so easily learned and tested for on a superficial verbal level, and a true understanding of mathematics, difficult both to produce and to recognize, is basic also to recent work on "creativity." E. Paul Torrance, in the short booklet entitled ..."Creativity" writes:[2]

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[1] Ibid., pp. 40, 41

[2] E. Paul Torrance, "Creativity." Department of Classroom Teachers, American Educational Research Association of the National Education Association. Research Pamphlet Series "What Research Says to the Teacher," #28. April 1963, p. 5.

2

> Many scholars have denied the possibility that young children can do productive thinking (producing something from what is cognized and remembered), and this has led to an overestimate of the child's receptivity. This misconception has led to an over-emphasis upon the importance of providing a stimulating environment to the neglect of providing a responsive environment, an emphasis upon recall and reproduction to the neglect of problem solving, creative thinking, and decision making.

The distinction is drawn from a different perspective, in relation to college education, by Torrance in the words:[1]

> Undergraduate college students have been known to produce almost all types of creative products such as inventions, medical discoveries, books, monographs, dramas, and operas. Usually, however, such accomplishments have been achieved outside of college requirements and sponsorship. Dissertations and theses, usually regarded as original contributions, tend to be evaluated in terms of correctness of methodology rather than in terms of originality, power, and worth of the ideas developed and tested.

Anyone who has confronted this distinction in modern education knows how remote and challenging is the peak of understanding, and how accessible and inviting are the lowlands of rote learning and knowledge of facts and techniques. Yet scaling this peak is the only worthy task of quality education.

This committment to achieve understanding constitutes one of the Project's essential working assumptions, and partially determines the nature of the "informal exploratory experiences."

The insistence upon the specific phrase "informal exploratory experiences" requires explanation. The assumption that children are empty vessels, unable to perform mathematically until they have been told what and how, and unwilling to do so except under threat of punishment or hope of external reward, dominates the thinking of a large number of teachers at every level of education. The activities involved in the Madison Project program involve such seemingly "natural" or "spontaneous" behaviour on the part of the children that some observers have assumed that no learning could be taking place. Many teachers -- as the operation of the tape panel has clearly revealed -- cannot comfortably allow such seemingly natural behaviour during mathematics classes. Time and again they intervene in order to show a child what to do, or how to do it, or to criticize, or to exhort.

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[1] Ibid., p. 7

This can destroy the learning atmosphere which is desired. In order to make such observers and teachers feel more comfortable with what is happening in the classroom, it has become necessary to emphasize that we are not here engaged in teaching mathematics lessons of the traditional sort, but rather ... "merely informal exploratory experiences." The distinctive name is necessary in order to reconcile people to dealing with a distinctive form of learning.

The underlying psychology is described excellently by Torrance:[1]

> In this writer's opinion, the weight of present evidence indicates that man fundamentally prefers to learn in creative ways -- by exploring, manipulating, questioning, experimenting, risking, testing, and modifying ideas. Teachers generally have insisted that it is more economical to learn by authority. Recent research suggests that many things, though not all, can be learned more effectively and economically in creative ways rather than by authority. It also appears that many individuals have an especially strong preference for learning creatively, learn a great deal if permitted to use their creative thinking abilities, and make little educational progress when we insist that they learn by authority. Such suggestions open exciting possibilities for better ways of individualizing instruction.
>
> Learning creatively takes place in the process of sensing problems or gaps in information, making guesses or hypotheses about these deficiencies, testing these guesses, revising and retesting them, and communicating the results. Strong human needs are involved in each stage of

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[1] Ibid., pp. 12 - 13

> this process. If we sense that something is missing or untrue, tension is aroused. We are uncomfortable and want to do something to relieve the tension. This makes us want to ask questions, make guesses, or otherwise inquire. Uncertain as to whether our guesses are correct, we continue to be uncomfortable. Thus, we are driven to test our guesses, correct our errors, and modify our conclusions. Once we discover something, we want to tell someone about it. This is why it is so natural for man to learn creatively. It seems so spontaneous that some people call it incidental learning.

The general nature of the "informal exploratory experiences" is indicated by Whitehead's remarks [1] that

> "A typical mathematical problem should be: Survey such and such a field, draw a plan of it to such and such a scale, and find the area."

and also [2]

> "Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible. The child should make them his own, and should understand their application here and now in the circumstances of his actual life. From the very beginning of his education, the child should experience the joy of discovery."

One "informal exploratory experience" -- dealing with average and variance, and used at the 6th grade level -- has been described earlier in this report.

In another "informal exploratory experience," children of about this age are asked to find the height of the school flagpole, which they can of course do in several ways, for example by sighting along a protractor to the top of the pole, making an accurate scale drawing, and using properties of similar triangles. (Alternatively, they can compare the shadow of the flagpole with the shadow of a person of known height, and again use properties of similar triangles. Yet again, if they know the trigonometric functions, they may make use of them.)

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[1] Whitehead, op. cit., p. 23

[2] Ibid., p. 14

In another "informal exploratory experience," 7th graders who are familiar with vector addition and the vector representation of forces are asked to predict the breaking or non-breaking of wool or nylon yarn under various conditions of loading with 10 and 20 pound lead weights.

Not all "informal exploratory experiences" are of an "applied" nature; most, in fact, are fairly abstract. In one instance, students (in grades 5 - 9) compile a long list of algebraic identities, and then seek to divide them into a set of axioms and a set of theorems, giving proofs for each of the theorems.

Additional examples of some of the learning activities developed by the Project are given in the report of the first year of this demonstration, (which bears the same title as the present report, but is dated 1962,) and many of them are shown in complete detail in the various films (cf. Section IV below.)

In general, the following attributes appear to be essential for good examples of "informal exploratory experiences:"[1]

1) The class activity is lead by the teacher, but it is not dominated by the teacher;

2) Where possible, children are given success experiences;

3) The classroom atmosphere is such that "failure is non-punitive";

4) The teacher attempts to present each task to the class with reasonable clarity;

5) The teacher does not show the children how to attack the problem;

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[1] Robert B. Davis, A Modern Mathematics Program as it pertains to the Interrelationship of Mathematical Content, Teaching Methods and Classroom Atmosphere. The Madison Project. 1962 report, p. 28.
Also cf. Torrance, op. cit., pp. 16-20.

6) The teacher is reasonably responsive to student suggestions as to methods of attacking the problem, and even as to alternative objectives and alternative activities;

7) The teacher may base appeals upon objective reality or upon the teacher's personal desires and preferences, but not upon authoritarian or formalistic criteria (e.g., the teacher may say "I'd rather do it the other way," or "Well, the other way looks like it might be easier;" he would not say "Now, how did I tell you to do this problem?");

8) The teacher formulates a flexible lesson plan in advance of the lesson, but he does not draw up a rigid lesson plan and attempt to adhere to it in spite of alternatives suggested by the students;

9) The teacher probably uses relatively informal language, and an informal manner;

10) The teacher listens to the children, and attempts to understand what they say. As Professor David Page and others have emphasized, it usually turns out that every student answer is at least partly correct, if one takes the trouble to identify the student's frame of reference.

11) The teacher treats the children with respect, not with condescension.

12) The teacher carefully avoids appearing omniscent; his statements nearly always suggest at least some measure of uncertainty in his own mind. Only in this way can he keep class activity directed toward the study of nature and of logical consequences, rather than toward authoritarian pronouncements.

It is possible for a program of "informal exploratory activities" to have a great cumulative effect; over a period of years there can be deep accumulated learning derived from such experiences.

"Informal exploratory experiences" may deal with relatively concrete matters (e.g., the use of Cuisenaire rods), or they may deal with quite abstract matters, such as

$$\square \times \square = {}^{-}4 \quad \text{or} \quad \square \times \square = 2.$$

IV. MATHEMATICAL CONTENT

The rationale behind the selection of mathematical content has been discussed elsewhere.[1] In this Section we list those mathematical experiences which have been shaped into reasonably definite (yet flexible) form, at the time of this writing, and those which are presently in preparation.

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[1] Cf. Robert B. Davis, "Report on the Madison Project," Science Education News (a publication of the American Association for the Advancement of Science,) December 1962, pp. 15-16; also, "Mathematical Thought and the Nature of Learning: The Madison Project View," Frontiers of Education. American Council on Education, (21 Audubon Avenue, New York 32, New York, 1963) pp. 79-83.

Grade Level	Topic	Presented in:	
		Film	Written Material
K-1	What do we mean by "3"? By 12? Etc. (Using colored washers)	Now in preparation	
K-5	Number names for Cuisenaire rods (Positive integers and fractions)	This follows the units developed by Caleb Gattegno, and distributed by Cuisenaire Corporation of America. Specific Madison Project films, etc., are now in preparation.	
1 or 2	What do we mean by "addition"? By "multiplication"? (Using colored washers)	Now in preparation	
1 or 2	Number line (pictorial representation of E_1)	1. A Lesson With Second Graders	1. Booklet which accompanies film "A Lesson With Second Graders"
2 - 9	Cartesian co-ordinates	1. A Lesson With Second Graders 2. First Lesson	1. Booklet which accompanies the film "A Lesson With Second Graders" 2. Booklet which accompanies the film "First Lesson" 3. Discovery in Algebra
2	Experience with Fractions (using kindergarten blocks)	Now in preparation	
2 - 6	Introduction of Signed Numbers ("Pebbles in Bag" Model)	1. A Lesson With Second Graders	1. Booklet which accompanies the film "A Lesson With Second Graders" 2. In-Service Teacher Training Package

Grade Level	Topics	Presented in:	
		Film	Written Material
2 - 9	Open Sentences (includes quadratic equations, use of signed numbers, inequalities)	1. A Lesson With Second Graders 2. First Lesson	1. Booklet which accompanies the film "A Lesson With Second Graders" 2. Booklet which accompanies the film "First Lesson" 3. Discovery in Algebra 4. In-Service Teacher Training Package
2 - 9	Uses of Monotonicity	Now in preparation	
3 - 7	Complete arithmetic of signed numbers	Now in preparation	1. Discovery in Algebra
2 - 9	Algebraic identities	1. Second Lesson	1. Discovery in Algebra
2 - 5	Concepts of Area, Perimeter, and Volume	Now in preparation	
3 - 9	Linear Graphs	1. Second Lesson	1. Discovery in Algebra
4 - 9	Non-Linear Graphs	1. Circles and Parabolas	1. Discovery in Algebra
4 - 9	Formal treatment of concept of "Variable"	Now in preparation	
4 - 6	Experience with Angles	Now in preparation	
4 - 9	Uniqueness, Implication, Contradiction (Based upon David Page's "Hidden Numbers")	Now in preparation	

Grade Level	Topics	Presented in: Film	Written Material
4 - 9	Identification of Functions		1. Matrices, Functions, and Other Topics
4 - 6	Derivation of Geometric Mensuration Formulae		1. Discovery in Algebra
4 - 9	Study of Empirically-Obtained Functions	1. Weights and Springs	1. Discovery in Algebra 2. Matrices, Functions, and Other Topics
4 - 9	General Form of Equations; General Solution	Now in preparation	1. Discovery in Algebra 2. Matrices, Functions, and Other Topics
4 - 9	Equivalent Equations		1. Discovery in Algebra
4 - 9	Arithmetic with Negative Digits, Non-Standard Names for Numbers, Non-Standard Algorithms for Subtraction, etc.	1. Kye's Arithmetic 2. The New Mathematics	Now in preparation
4 - 9	Simultaneous Equations		1. Discovery in Algebra
5 - 9	Role of Axioms and Theorems in Algebra	1. Axioms and Theorems 2. Theorem: □ + □ = 2 × □ (3. Tape Recording #D-1)	1. Discovery in Algebra
5 - 9	Addition and Multiplication of Matrices	1. Matrices	1. Booklet which accompanies the film "Matrices" 2. Matrices, Functions, and Other Topics

Grade Level	Topics	Presented in:	
		Film	Written Material
5 - 9	Programming Digital Computers	Now in preparation	
5 - 9	Complex Numbers in relation to Matrices	1. Solving Equations with Matrices 2. Complex Numbers via Matrices	1. Matrices, Functions, and Other Topics
5 - 9	Word Problems		1. Discovery in Algebra
6 - 9	Average and Variance	1. Average and Variance	
6 - 9	Truth Tables and Inference Schemes	Now in preparation	
6 - 8	Similar Triangles; Determination of Inaccessible Distances	Now in preparation	
6 - 9	Extension of Number Systems	1. Complex Numbers via Matrices	1. Matrices, Functions, and Other Topics 2. Limits
7 - 9	Trigonometric Functions Introduced via Oscilloscopes and Numerical Tables	Now in preparation	
7 - 9	Vectors and Forces	1. The New Mathematics 2. Forces and Vectors	
7 - 9	Velocity and Acceleration	Now in preparation	
7 - 9	Limit of a Sequence	Now in preparation	

Grade Level	Topics	Presented in:	
		Film	Written Material
9	Logical Operators as Mappings from Cartesian Products of Truth Value Spaces	Now in preparation	
6 - 9	Summation Notation	Now in preparation	
7 - 9	Finite Difference Methods	Now in preparation	
9	Areas and Volumes as Limits of Sums	Now in preparation	

V. BEHAVIOURAL SCIENCE ASPECTS

Although it is concerned with instruction in mathematics, the Project is not solely a matter of mathematics. Various behavioural science matters evidently enter into the picture.

As indicated earlier in this report, these behavioural science aspects are divided into four categories:

As a rough indication of where such a K – 8 program might lead, one might think of a 9th and 10th grade algebra and geometry course following the general lines of Moise's Elementary Geometry from an Advanced Viewpoint. (In addition to geometry, this volume contains an excellent treatment of algebra.)

i) The "Description Problem"

iii) Identification of Working Hypotheses

iv) Consideration of Relevant Matters Which Lie Largely Outside of Project Control

1. The "Description Problem." What are some of the essential features of the "informal exploratory experiences" upon which the Project's program is largely based? Answering this question constitutes the "description problem."

The kind of answer desired is an important question. The Project seeks an answer consisting of three parts, at least:

i) films showing actual classroom lessons, by means of which viewers can see for themselves every possible visual and auditory aspect of typical "informal exploratory experiences," including behaviour of teachers and of students;

ii) tape recordings which record auditory aspects of the "informal exploratory experiences," omit visual aspects, but are otherwise similar to films;

iii) written descriptions and analyses, which attempt identification of significant aspects of "informal exploratory experiences" in abstract verbal terms.

At the present time, the films are the most complete part of this reporting, and must be viewed by anyone who wishes to understand what the Project means by "in-

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ii) The "Quality Control" Program

iii) Identification of Working Hypotheses

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At the present time, the films are the most complete part of this reporting, and must be viewed by anyone who wishes to understand what the Project means by "informal exploratory experiences."[1] Further reporting and description via all three media continues, and will continue during the coming year.

2. The Quality Control Program. This has been outlined in some detail in the preceding report submitted to the Commissioner of Education. It presently consists of two parts: an intensive study of some schools and some classes, including considerable use of clinical interview techniques, and a broad (but necessarily more abstract and more superficial) study of students, teachers, and parents in all schools directly related to the present demonstration.

The quality control program is directed by Mr. Herbert Barrett of Weston, Conn.

A written report on the results of this program will be prepared at the end of the 1963-1964 academic year.

3. Identification and Consideration of Working Hypotheses. This aspect is divided into two parts: working hypotheses as stated by Project personnel, and apparent assumptions as perceived by outside personnel viewing Project activities -- that is to say, what the Project says, and what it appears to do.

Stated working assumptions fall into four categories:

i) Assumptions about the nature of mathematics. Project personnel say that mathematical thought consists of at least the following components:

The ability to discover pattern in abstract situations.

The ability to use independent creative explorations to extend "open-ended" mathematical situations.

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[1] For written materials, compare the previous report submitted to the Commissioner of Education, U. S. Department of Health, Education and Welfare (dated 1962), the booklet "The Madison Project -- A Brief Introduction to Materials and Activities," and the Science Education News, 12 - 62 American Association for the Advancement of Science "The Madison Project" pp. 15 and 16.

The possession of a suitable set of mental symbols that serve to picture mathematical situations in a pseudo-geometrical pseudo-isomorphic fashion, somewhat as described by Polya, Leibnitz, and the psychologist, Tolman.

A good understanding of fundamental mathematical concepts, such as variable, function, Cartesian co-ordinates, open sentence, truth set, matrices, implication, contradiction, axioms and theorems, uniqueness, mapping or transformation, linearity and proportionality, etc.

Reasonable mastery of important techniques.

Knowledge of mathematical facts.

It should be noted that this list markedly distinguishes Project objectives from those of most "traditional" mathematics curricula; indeed, traditional curricula were concerned almost exclusively with the last two items on the list above.

In addition to this list of "intellectual" ingredients, the Project assumes that good instruction must seek to develop certain "behavioural" characteristics (in the area of values, attitudes, etc.), such as:

A belief that mathematics is discoverable.

A belief in one's own ability to discover mathematics.

A recognition of the "open-endedness" of mathematics.

Honest and wise self-critical ability.

A commitment to the value of abstract rational analysis.

Recognition of the role of educated intuition.

A feeling that mathematics is "exciting" or "fun" or "challenging" or "rewarding" or "worthwhile."

Given these assumptions about the goals of mathematical study, it is clear that one must reject the traditional emphasis on telling students what to do and on rote drill.

ii) Assumptions about the process of learning. Project personnel say that they are concerned with the utilization of the child's natural modes of learning: as, for example, in learning to talk, to find his way around his own neighborhood, etc. The general view of learning stated is similar to the picture of learning which emerges from the following:

Brecher, Ruth and Brecher, Edward. "Gifted Children Need Freedom to Learn," *Parents' Magazine*. (Bergenfield, New Jersey), June 1962, pp. 44 ff.

Bruner, Jerome S. *The Process of Education*. Cambridge, Massachusetts: Harvard University Press.

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Courtis, Stuart A. "Our Choice: Revolution or Destruction," *Educational Leadership*. Vol. 20, No. 8, May 1963, pp. 520-522.

Cronbach, Lee J. *Educational Psychology*. New York: Harcourt, Brace and Co., 1954.

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Torrance, E. Paul. "Creativity." Department of Classroom Teachers, American Educational Research Association of the National Education Association. Research Pamphlet Series "What Research Says to the Teacher," #28. April 1963.

Whitehead, Alfred North. Aims of Education. (Mentor Books) New York: Macmillan Company, 1929.

This same general position has also been described in unpublished discussions by Robert A. Rosenbaum (Wesleyan University), W. W. Sawyer (also of Wesleyan), Omar K. Moore (Yale University), Layman Allen (Yale University), John Downing (University of London), and Douglas McGregor (M.I.T. and Antioch), among many others.

iii) Assumptions about School and Classroom Organization. These follow generally the pattern of the books and articles listed immediately above. In particular, the Project is concerned with active participation by students (which may be physical or may be purely mental, but which is not passive as in memorizing, reading, and listening; it may consist of thinking through new problem solving situations, critically analyzing another student's work, etc.), and with classroom social organization consistent with lively student participation, and with considerable freedom for students.

The Project's working hypotheses as represented in actions and perceived by outside observers have been reported especially in the area of practices related to motivation.

Project classes show far less use of external reinforcement schedules (e.g. praise, exhortation, threats, etc.) than appears to be common in contemporary American education at corresponding grade levels.

Instead, the Project appears to rely heavily upon activities which are intrinsically rewarding.

An activity might be intrinsically rewarding while it is in progress (i.e., because the process is rewarding) or because it leads to a desired result (i.e., the product is rewarding). Such a distinction may or may not prove viable, but studies of motivation oriented in this direction are being undertaken by the psychologists Carl Pitts and Richard de Charms, in relation to the Madison Project's apparent uses of motivation.

The Project's stated working assumptions on the nature of data collection, analysis, and reporting are also important. A fuller discussion of these is planned for future release, but a few general remarks deserve inclusion here:

For one thing, the Project is aware of several undesirable aspects of using achievement tests.

a. It is felt that, in present middle-class American society, adults generally (and teachers in particular) are often prone to make too many evaluative judgments on children. Such judgments do not serve to further creative growth of the child; they have the additional undesirable property of impairing a mutual respect between child and adult. The Project has attempted to develop a supplementary mathematics program in which the child's accomplishments are acknowledged occasionally, and are generally appreciated and respected, but neither the child nor his work is exposed to frequent judgments by anyone other than the child himself.

b. Frequent use of tests or other evaluative apparatus can create an expectation of a "normal" rate of progress, and can interfere with the educational ideal of highly individualized learning, with each child progressing at his own pace.

c. Tests, when prominently used, can focus upon an assumed specific educational destination, whereas the Project wishes to regard the learning of science and mathematics as a growth process, an exploring of roads whose end points are never known a priori. Indeed, there are no "destinations," since each road leads only to yet other roads to be explored.

As Whitehead has written, the scientist does not discover in order to know, he knows in order to discover. The Project is currently attempting to construct tests where a student's performance in learning some new material can be used as a means for assessing his previous learnings. The film Matrices shows such a task in a group situation.

To turn to a second matter, the Project assumes that science is not necessarily committed to the measurement of numerical variables. The assumption of such a commitment seems to be a ubiquitous attribute of contemporary philosophical thought, but a careful consideration of scientific theory does not support such a position. Modern physics, for example, is much concerned with symmetry, with group theory, with linear operators on vector spaces -- none of which is directly or essentially a matter of numerical variables.

On the contrary, science is concerned primarily with the development of a suitable set of concepts for thinking about reality in relation to some suitable abstract models. Numerical variables may or may not be involved, and their prominent presence should not be assumed to confer a higher status than their conspicuous absence.

The Project can consider with equanimity a process of "research" which never at any time has reference to any numerical variable, nor to any process of measuring or otherwise determining values for any numerical variable.

4. Matters Largely Beyond Project Control. At the present time, the Project's main concern in this area is with the apparent deterioration of student performance at about the 7th and 8th grade level. This phenomenon has been observed and noted in unpublished discussions by most of the "new curriculum" studies which deal with both elementary and junior high school age children. A decline in creativity, as measured by "creativity" tests, at approximately this age level is reported by Torrance.[1] At least some of this decline appears to be related to the present culture in the United States, and is not observed in the same form elsewhere (cf. Torrance[2]).

Because of the Project's interest in following the same students for many years, the apparent decline of student performance generally (whether involved in new curricula or not) at the junior high level poses serious problems in assessing the effectiveness of previous instruction on a cumulative basis.

VI. PURPOSES AND EFFECTIVENESS

The Madison Project in general, and the present "demonstration" in particular, is conceived as an exercise in planned evolution. The general effectiveness of Project work must then be assessed partly in relation to two questions:

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[1] Torrance, ibid. pp. 16-17

[2] Ibid.

i) What evidence is there that the Project is helping to produce changes in the teaching and learning of mathematics, in the specific directions discussed above?

ii) What evidence is there that these specific directions are reasonably appropriate ones?

In short: Are we moving? Are we headed in a reasonable direction?

The rate of change referred to in question i) need not -- indeed, probably should not -- be precipitate. Yet it should be great enough to represent genuine change. An exercise in evolution which produces no evolution must be judged a failure.

One hundred and seventy-five classes of students, or approximately 5,000 individual students, in a nation with approximately 10^6 teachers and about 10^6 classes (at these grade levels) is surely not precipitate haste on a national scale. Probably the scale of such a demonstration is somewhat too small for real impact, not to mention for real continuity.

In fact, a larger number of classes are actually involved, as will be revealed in next year's report, if improved reporting procedures can be developed. Because of the Project's extensive programs of teacher training and dissemination of information, many more schools are actually working with the Project than are represented by present reporting procedures.

As additional measures of impact, one might note that:

i) Newspaper and magazine coverage of the Project's work has been extensive, and has involved unusually full and careful reporting. Publications involved have included The Christian Science Monitor, The New York Herald Tribune, The Saturday Evening Post, The St. Louis Post-Dispatch and The PTA Magazine, and Life Magazine's volume entitled Mathematics.

ii) Within professional literature, the Project has been discussed in The Arithmetic Teacher, The Mathematics Teacher, The New York State Mathematics Teachers' Journal, Science Education News, and the American Mathematical Monthly, among other places.

iii) A half-hour TV show was presented on WNBC-TV, Channel 4, New York City, under the sponsorship of the National Education Association.

iv) Volume of Material: For the past several months, Project mailings have been on the average of 1,000 pieces of written material per week. Some films are so popular that between 30 and 40 prints of a single title (in use simultaneously) are required to meet demands. Relatively unpopular publications of a relatively advanced nature, intended for teachers rather than students, sell at the rate of about 1,000 per year. Additional sets of films are located in England, West Africa, East Africa, and Australia, where they are widely used.

v) Visitors: Vistors have come from as far away as East Africa specifically to observe the Project; visitors have come from Canada for this purpose, from the Dominican Republic, and all the way across the United States (e.g., from Seattle to Weston, Connecticut). Many come half-way across the United States for this purpose. As many as 50 or 60 visitors may be at Weston on the same day.

vi) Invited addresses, courses, and workshops: By invitation, the Project has offered courses and workshops during the past year at Webster College, Syracuse University, Washington University, the University of Kansas City, the University of Rochester, Harvard University, Tulane University, Earlham College, Antioch, Minot (North Dakota) State Teachers College, the University of Arizona, the College of Idaho, Boston College, and elsewhere, and invited lectures and workshops at professional meetings in Northern California, in Oklahoma, in Missouri, in Wisconsin, in Illinois, in Pennsylvania, in Massachusetts, in Washington, in Indiana, in Ohio, in Colorado, and elsewhere. Project personnel have traveled extensively -- by invitation -- in England and on both coasts of Africa.

The second question is perhaps somewhat easier to answer, although in matters of planning for the future no person nor any group of people are omniscient. The Project tries to report to interested professional people -- mathematicians, scientists, engineers, psychologists, psychiatrists, educators, teachers, etc. -- as fully as possible, especially via films, and in such a way as to maintain a dialogue, with information and opinions flowing in both directions. Among the forums for such dialogues, these are especially significant:

i) the meeting on "Learning About Learning" conducted by Professor Jerome Bruner at Harvard University, during the summer of 1963;

ii) the Cambridge Conference on School Mathematics meeting at Radcliffe College during the summer of 1963, under the leadership of Professor Andrew Gleason of Harvard University;

iii) the Elementary School Science Project, meeting in Cambridge, Massachusetts, during the summer of 1963, under the direction of Professor David Hawkins of the University of Colorado;

iv) the Conference on Educational Problems of Culturally Deprived Students, meeting at M.I.T.'s Endicott House during the summer of 1963, under the leadership of Professor Jerrold Zacharias of M.I.T.

The Project has participated actively from the outset in E.S.I.'s important program on African Education.

A careful study of the report of the Cambridge Conference will reveal that the Project's activities are notably consistent with the Conferences' recommended directions for evolution over the next thirty years.

In addition to many eminent professional people involved in the preceding activities, the Project has enjoyed the counsel of several others, notably Professor Marshall Stone and Professor Saunders MacLane, both of the University of Chicago. The Project does not claim the endorsement of any of these individuals or groups, but the items reported above indicate that a significant two-way dialogue is maintained between the Project and an important sector of the American academic community. The hope is that, in the future, even this sizeable exchange can be expanded further.

VII. SUMMARY

The present "demonstration" is a part of the over-all activity of the Madison Project. This over-all activity is aimed at giving children in schools an experience of mathematics in the creative and open-ended form in which it is known to mathematicians. The present "demonstration" can be divided into several phases. Phase I is concerned with introducing into schools a sequence of "informal exploratory experiences" dealing mainly with modern axiomatic algebra, with co-ordinate geometry, and with the study of functions; Phase I is also concerned with the articulation of these "experiences" into a powerful cumulative sequence. Phase II is concerned with an integration of these "experiences" into the over-all arithmetic (and possibly science) programs of the schools. In Phase II the sequence of "informal exploratory experiences" provide a strong foundation of basic mathematical concepts, from which arithmetical matters can emerge as "applications" and "special cases." Although there is no sharp division, the demonstration is now entering Phase II.

Many complex problems of research strategies and of behavioural science are involved; these are discussed in some detail in the preceding report, and also elsewhere.

It is hoped that this "demonstration" can help to map out a rational evolutionary path for the mathematics curriculum over approximately the next three decades or so.

Teacher training, and other, activities are co-ordinated with the present "demonstration." Important aspects of Project work are reported via films, and these films should be regarded as an integral part of the present written report. One might view the films as supplements to the present report, but it would probably be more accurate to regard this written report as supplementary to the films. They are the basic report.

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A Lesson With Second Graders
First Lesson
Second Lesson
Circles and Parabolas
Weights and Springs
Kye's Arithmetic
Education Report: The New Math
Axioms and Theorems
Theorem: $\square + \square = 2 \times \square$
Matrices
Solving Equations with Matrices
Complex Numbers via Matrices
Average and Variance
Forces and Vectors

Tape Recordings[1]

#D-1

Films and Written Materials in Combination[1]

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[1] Available from: Madison Project, Webster College, St. Louis 19, Missouri.